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A clinical–experimental simulation of changes in intercanine width associated with the correction of crowding: a pilot study

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SUMMARY The purpose of this article is was to introduce a mathematical model, investigating the anticipated changes in intercanine width associated with the resolution of crowding, using the leeway space. A linear regression formula was computed to predict the change per millimetre of 'intercanine tip space deficiency'.

After digitizing 61 mandibular casts, 14 co-ordinates per model were used to fit an 'individualized optimal curve' consisting of either a hyperbolic cosine function or parabola anteriorly and a third degree polynomial function bilaterally. The total amount of tooth material was measured between the cusp tips of the canine teeth and subtracted from the available intercanine tip arch length to render the intercanine tip space deficiency. This value was then used to allocate patients to an experimental [31 subjects, 14 boys and 17 girls, mean age 11.7 years, standard deviation (SD) 1.4 years] or a control (30 subjects, 10 boys and 20 girls, mean age 11.6 years, SD 1.1 years) group. After digitally aligning the incisors and canines on the selected optimal curve, the new intercanine width was calculated.

Significant differences ($P < 0.001$) in intercanine expansion were found between both groups (control group: mean –0.07 mm, SD 0.31 mm; experimental group: mean 1.13 mm, SD 0.51 mm) due to a clinically significant increase of approximately 0.6 mm in intercanine width (y) per millimetre of calculated intercanine tip space deficiency (x), leading to the following regression formula: $y = 0.09 + 0.52x$.

Introduction

Crowding is the practical expression of a tooth size–arch length discrepancy: the discrepancy between the accumulated mesiodistal tooth width of mainly the anterior dentition and the space available to accommodate these teeth in an aligned fashion, leading to rotations and buccal or lingual displacements of these teeth.

One possible approach to the correction of crowding is using the leeway space (Nance, 1941), which was shown by Dugoni *et al.* (1995) to be relatively stable. Nine years after treating patients with crowding using only a lingual arch, they found acceptable incisor alignment in 76 per cent of patients [Little irregularity index (LII): 2.65 mm, standard deviation (SD) 2.09 mm], which was only half that observed in a study by Little *et al.* (1990), in which a lip bumper was used to gain more than 0.5 mm of space per side (LII: 6.06, SD 2.79 mm). Brennan and Gianelly (2000) calculated that approximately 61 per cent of patients can be treated non-extraction when using the average 4.4 mm of combined bilateral leeway space. Those authors stated that when the mandibular molars were minimally distalized (<0.5 mm per side), and the intercanine width expanded 1 mm, this figure could increase up to 76 per cent. Of course, the question arises whether any additional increase in intercanine width would be acceptable after moving the canines distally and buccally into the leeway space, since most studies agree that above a 1 mm increase in this dimension, a return to the pre-treatment value is found in the long term (Burke *et al.*, 1998).

Basically, in any situation were crowding is corrected without extensive interproximal stripping, intercanine expansion would be expected due to the horseshoe shape of the mandibular alveolar process. This was indirectly demonstrated by Noroozi (2000), upon proposing a formula to calculate the amount of canine retraction needed for the correction of lower incisor protrusion, using simple mathematics. Even though in the study by Dugoni *et al.* (1995), only a passive lower lingual arch was used to correct crowding (preserving the leeway space), they found an increase in intercanine width of 2.41 mm (SD 1.80 mm). As an alternative, providing space for the correction of crowding by extracting lower first premolars, Gardner and Chaconas (1976) reported a 1.92 mm increase in intercanine width (SD 2.08 mm). In fact, although (unintentional) intercanine expansion has been reported frequently after orthodontic treatment (Burke *et al.*, 1998), little is known about what fraction of this expansion is secondary to the correction of crowding, with resulting distal/lateral movement of the canines. In consequence, it is also difficult to ascertain what part of the observed expansion is actually 'therapeutically induced' (any additional expansion). Also, in treatment planning, these changes in intercanine width due to the correction of (severe) crowding seem to have been disregarded, probably because little is known about this aspect of treatment. Nevertheless, if it is preferred, for reasons of stability, not to increase the intercanine width more than 1 mm (Burke *et al.*, 1998), it would be interesting to pre-determine whether this is feasible at all using the leeway space to correct crowding, or whether interproximal stripping would be advisable to limit this increase.

It was therefore the aim of this study to investigate the changes in intercanine width associated with the retraction of the canines into the leeway space, during the correction of crowding. Using regression analysis, a formula was constructed that should enable the prediction of this change per millimetre of 'intercanine tip space deficiency'. As the null hypothesis, it was assumed that distal movement of the canines during the correction of crowding using the leeway space does not lead to a significant increase in intercanine width.

Subjects and methods

In general terms, in order to provide an answer to the present research question, the methodology would have to include six major steps:

1. The pre-treatment intercanine width is measured.
2. An individual patient arch form is constructed. This step requires the models to be digitized.
3. The resulting arch form would be used to determine the pre-treatment anterior space deficiency.
4. It would also serve to 'simulate' the correction of crowding by 'aligning' the incisors' mesial and distal contact points on the curve, as well as the mesial anatomic contact point at the canine and cusp tip.
5. This would enable the measurement of the new, 'aligned' intercanine width.
6. Applying this procedure to a large sample of patients, and plotting the space deficiency against the intercanine expansion, would allow the construction of a regression formula in order to predict the change in intercanine width resulting from the correction of the anterior space deficiency.

A control group would be necessary to confirm that the observed effect is a consequence of the correction of the space deficiency and not an artefact resulting from the chosen methodology.

Therefore, 61 subjects, 24 males and 37 females (mean age: 11.7 years, SD: 1.3 years), were selected from three private orthodontic practices using the following inclusion criteria: all permanent incisors, canines, and first premolars fully erupted. In patients exhibiting crowding, the second primary molars should be present or should have recently exfoliated. The incisors exhibit a fairly normal buccolingual inclination, according to visual inspection. There is no clear ectopic or tipped position of the canines. There are no definitive teeth in crossbite. The occlusion is fairly symmetric. Patients had not undergone any previous orthodontic treatment. The casts are in good condition.

All lower models were trimmed such that, when placed with the occlusal surface on a flat plane, the first molars and incisors were in contact with the table. A reference plane was thus constructed through the distobuccal cusp tips of the first molars and the incisors. All changes were measured in relation to this reference plane. Subsequently, 16 points were marked with a Rotring rapidograph 0.18 fine ink pen (Figure 1).

The models were then placed approximately in the middle of the scanning surface of a HP scanjet 8200 desktop scanner (Hewlett Packard, Palo Alto, California, USA), on top of a transparent calibration sheet. Paperweights added in the x and y direction insured the calibration sheet was pushed flat against the scanner surface. Scanning was performed at a resolution of 2400 dpi.

The images were then imported into a digitizing software program (DigitizeIt 1.5.7, I. Bormann, Bormisoft, Germany), in order to identify the co-ordinates of the 16 landmarks. Since scanning was performed at 2400 dpi, the resolution of the digitizing procedure was 0.011 mm. The generated co-ordinates were exported to Excel (Microsoft Corporation, Redmond, Washington, USA) to orientate the cluster of points and calculate the curve fitting parameters (Figure 2). The resulting values were subsequently entered into Findgraph (Findgraph for Windows, version 1.482, UNIPHIZ Lab, 2001-2004, www.findgraph.com), a curve fitting and graphing program. The author of this software program compiled a dedicated plug-in, automating the curve fitting procedure.

Curve construction has been described more in detail elsewhere (Wellens, 2007). In brief, the mandibular arch form was described using three different functions linked together (Hnat *et al.*, 2000): anteriorly, in between the canine tips, either a parabola or a hyperbolic cosine function was used. Although both functions are only slightly different, the latter tends to be slightly broader. Laterally, a third order polynomial was fitted. This is a function of the form $y = ax^3 + bx^2 + c$, where a, b, and c are numbers, and the third order refers to the highest power of x, used in the function.

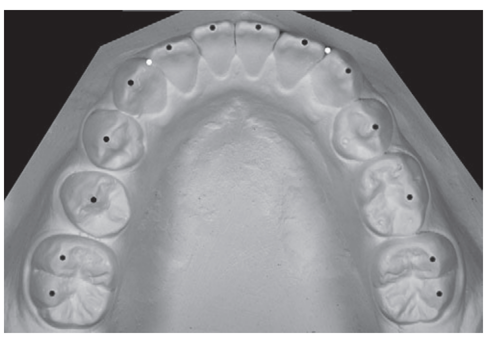

Figure 1 Digitized landmarks: the middle of the incisal edge of the central and lateral incisors; the cusp tip of the canines; the buccal cusp tip of the first premolars; the midbuccal cusp tip of the second primary molar or buccal cusp tip of the second premolar; the mesiobuccal and midbuccal cusp tip in case of three buccal cusps, or the mesiobuccal and distobuccal cusp tip in case of two buccal cusps, of the first definitive lower molars; the mesial anatomic contact point of the canines.

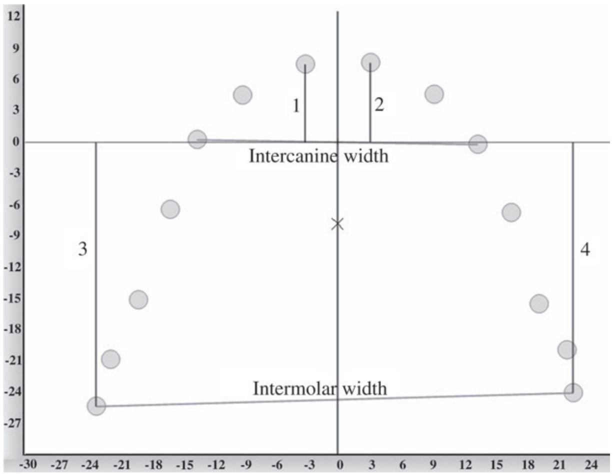

Figure 2 Curve fitting parameters: Anterior arch depth = (1 + 2)/2; Posterior arch depth = (3 + 4)/2; Total arch depth = anterior arch depth + posterior arch depth.

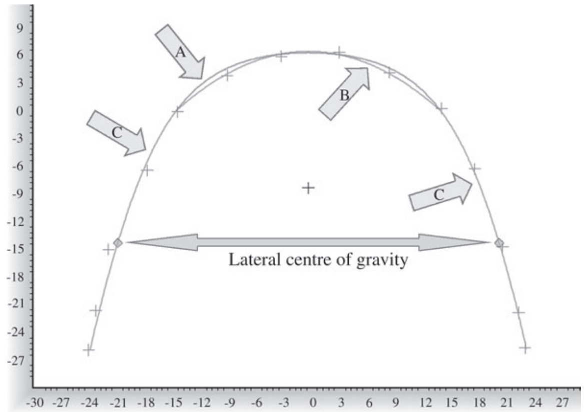

Figure 3 Functions used for curve fitting: Anterior functions: A. hyperbolic cosine function: $y = (-\cosh(x/(ICW/2))\cosh(PAD + 1)) + 1 + PAD)f$ where, ICW = intercanine width, PAD = posterior arch depth and f = curve fitting factor, calculated by Findgraph B. parabola: $y = i\,(fx^2 + gx + h)$ where, f, g, h, and i are curve fitting factors, calculated by Findgraph. Lateral functions: C. Polynomial: $y = fx^3 + gx^2 + hx$, or alternatively $y = x\,(fx^2 + gx + h)$ where, f, g, and h are curve fitting factors, calculated by Findgraph.

The lateral function was constructed in between the averaged bilateral positions of the canine tip, the first molar mid or distobuccal cusp tip (depending on the presence of three versus two buccal cusp tips, respectively), and a symmetric lateral centre of gravity (Figure 3). This was undertaken to allow flexibility in the description of the lateral arch curvature. Curve construction was thus performed preserving the intercanine and intermolar width, as well as arch depth (Felton *et al.*, 1987; Hnat *et al.*, 2000). The curve fitting parameters were calculated using the least squares method.

The optimal anterior function was selected by calculating the average perpendicular distance of the points to the curve, that is, the perpendicular distance of each point to the curve was measured, and the resulting distances were averaged. The curve with the lowest mean value was selected. Using either a parabola or hyperbolic cosine function anteriorly, an attempt was made to accommodate rounded as well as more tapered anterior arch forms. By stringing together the anterior and two lateral segments, the ‘optimal’ individual arch form was defined, after which correlation calculations could be performed between the resulting function and the digitized points (see ‘Statistical analysis’).

The mesiodistal width of the four lower incisors was then measured, using a digital sliding calliper with finely ground tips up to a precision of 0.01 mm. The calliper was held perpendicular to the long axis of the incisor, exerting minimal force to prevent the chipping off of material. Each measurement was repeated five times, and the average was calculated.

Allocation of patients to the control or experimental group was performed by calculating the arch length along the selected anterior curve in between the averaged canine tip positions. By summing the tooth widths of the four lower incisors and the distances between the mesial anatomic contact point of the canines and their respective cusp tips, the total intercanine tip tooth width was known, which could be subtracted from the intercanine tip arch length value calculated earlier. The resulting difference was defined as the calculated intercanine tip space deficiency and was utilized to allocate patients to either the control (deficiency < 0.5 mm) or experimental (deficiency ≥ 0.5 mm) group. This led to an experimental group containing 31 subjects: 14 males and 17 females (mean age 11.7 years, SD 1.4 years), and a control group containing 30 subjects: 10 males and 20 females (mean age 11.6 years, SD 1.1 years).

Finally, all anterior teeth were digitally aligned on the individualized arch form. Starting from the midline, the mesial and distal anatomic contact points of the incisors were placed on the curve. After aligning the mesial anatomic contact point and tip of the canine in the same way, the new intercanine width, intercanine expansion, and mesio-distal shift of the canine tip could be calculated.

Error study

In order to establish the repeatability of the digitizing procedure, all points were re-digitized for 10 randomly selected patients, rendering 140 consecutive measurements. Using Wilcoxon’s signed rank test on the difference between the original and repeated x and y co-ordinates separately, revealed no significant differences (x co-ordinates: mean difference 0 mm, SD 0.011 mm, y co-ordinates: mean difference 0 mm, SD 0.011 mm).

For the same 10 patients, anterior tooth widths were re-measured, at least 2 weeks apart. The combined value for the six anterior teeth were compared (original versus repeated measurement, including the redigitized values for the distance: canine tip-mesial anatomic contact point) using a two-sample t-test, after testing for normality using the Shapiro–Wilk test. No significant difference was found (mean 0.063 mm, SD 0.1 mm).

Statistical analysis

All tests were performed using the Statistical Package for Social Sciences (SPSS Inc., Chicago, Illinois, USA, version 12.0.0 for Windows). Significance was predetermined at the 0.05 per cent level of confidence. Intra- or inter-group comparisons were performed using either t-tests or Mann–Whitney U-tests, depending on Levene's test to confirm homogeneity of variance, and the Shapiro–Wilk test to assess normality of the distribution. To assess the 'curve fit', Pearson's correlation coefficients were calculated between the 14 digitized incisal/occlusal landmarks used earlier for curve construction (Figure 1) and the resulting curve. Since the least squares method, used for curve fitting, assumes all the variability is located in the y co-ordinates, this means the y_1 value of every digitized point with co-ordinates (x_1,y_1), is correlated to the y_2 value of the corresponding point on the curve (x_1,y_2) (Wellens, 2007). Also, the average perpendicular distance of all digitized points to the curve was calculated.

Results

The results of the intra-group comparison for the control and experimental groups are presented in Table 1. As no significant differences were found, the data for boys and girls were pooled for further analysis. Inter-group comparison demonstrated a highly significant difference in anterior arch depth and in calculated intercanine tip space deficiency (Table 2).

When comparing curve fit using Pearson's correlation coefficients, high coefficients were found in both the control and the experimental group (Table 3). Not surprisingly, coefficients were slightly but significantly higher for the control group in comparison with the experimental group ($P < 0.05$). Alternatively, for the averaged perpendicular distance of the digitized points to the curve, a similar difference was observed; for patients in the control group, lower values were generally found in comparison with the experimental group ($P < 0.01$, Table 3).

After aligning the mandibular anterior teeth, a significantly larger intercanine expansion was found in the experimental group compared with the control group ($P < 0.001$, Table 3). In addition, the canine tip tended to shift distally more in the experimental than in the control group ($P < 0.001$, Table 3).

The following regression formula was computed for intercanine expansion versus intercanine tip space deficiency:

$y = 0.09 + 0.52x$ (Figure 4), standard error of the estimate: 0.196.

Discussion

Many different approaches have been proposed when trying to mathematically describe the human arch form (Wellens, 2007). Reported methods range from the use of simple geometric forms such as the parabola (Jones and Richmond, 1989), and ellipse (Currier, 1969), to geometric functions such as catenary curves (Pepe, 1975; Germane *et al.*, 1992), polynomials (Pepe, 1975; Noroozi *et al.*, 2001), cubic spline functions (BeGole, 1980), conic sections (Sampson, 1981), the beta function (Braun *et al.*, 1998), combinations of the hyperbolic cosine function and betafunction (Hnat *et al.*, 2000), and Fourier analysis (Lestrel *et al.*, 2004). Most studies used well-aligned dentitions to test the descriptive method of interest. However, the applicability of the currently employed method of curve description to late mixed dentition patients with crowding has recently been demonstrated (Wellens, 2007).

Table 1 Intra-group comparison of males and females for the control and experimental group.

Control group data	Males (n = 10)		Females (n = 20)			Pooled (n = 30)	
Curve fitting parameters	Mean	SD	Mean	SD	P	Mean	SD
Intercanine width	26.90	1.13	25.85	1.52	ns	26.20	1.47
Intermolar width	46.99	2.59	46.25	2.08	ns	46.49	2.24
Anterior arch depth	6.07	1.12	5.92	1.17	ns	5.97	1.13
Posterior arch depth	25.58	1.48	24.37	1.98	ns	24.77	1.90
Total arch depth	31.65	1.83	30.29	2.69	ns	30.74	2.49
Calculated space deficiency	−0.36	0.65	−0.16	0.44	ns	−0.23	0.52
Experimental group data	Males (n = 14)		Females (n = 17)		P	Pooled (n = 31)	
Curve fitting parameters	Mean	SD	Mean	SD		Mean	SD
Intercanine width	25.42	1.69	25.66	1.55	ns	25.55	1.59
Intermolar width	47.37	1.84	46.16	2.31	ns	46.70	2.17
Anterior arch depth	4.97	1.17	4.54	1.19	ns	4.74	1.18
Posterior arch depth	25.81	1.59	25.16	1.85	ns	25.45	1.74
Total arch depth	30.78	2.41	29.70	1.62	ns	30.19	2.05
Calculated space deficiency	2.23	1.41	1.67	0.82	ns	1.93	1.14

ns, Not significant.

Table 2 Inter-group comparison of curve fitting parameters.

Curve fitting parameters	Control group ($n=30$)		Experimental Group ($n=31$)		P
	Mean	SD	Mean	SD	
Intercanine width	26.20	1.47	25.55	1.59	ns
Intermolar width	46.49	2.24	46.70	2.17	ns
Anterior arch depth	5.97	1.13	4.74	1.18	***
Posterior arch depth	24.77	1.90	25.45	1.74	ns
Total arch depth	30.74	2.49	30.19	2.05	ns
Calculated space deficiency	−0.23	0.52	1.93	1.14	***

***$P < 0.001$.
ns, Not significant.

Table 3 Inter-group comparison of post-simulation results.

Results	Control group ($n=30$)		Experimental group ($n=31$)		P
	Mean	SD	Mean	SD	
Measured space deficiency	−0.08	0.51	2.15	1.18	***
Intercanine expansion	−0.07	0.31	1.13	0.54	***
Average vertical shift canine tip	−0.04	0.41	1.79	1.12	***
Correlation points/curve	0.995	0.004	0.991	0.008	*
Average perpendicular distance points/curve	0.32	0.10	0.49	0.20	**
Signed average perpendicular distance of the points to the curve	0.01	0.07	0.08	0.14	ns

*$P < 0.05$; **$P < 0.01$; ***$P < 0.001$.
ns, Not significant.

The resulting correlation coefficients between the curve and the digitized points were quite high (Table 3), with a fairly low corresponding average perpendicular distance of the points to the curve in both groups, indicating the proposed method was successful at describing arch form for the control, as well as the experimental patients.

When aligning the incisors and canines on the constructed curve, a highly significant increase in intercanine distance was found (Table 3), indicating that when the natural arch form, arch width, and depth are respected, the canines will, in theory, move buccally to a statistically and clinically significant degree, when distalized into a wider part of the alveolar process. The linear regression formula for the combined results of the control and experimental group suggests that for every millimetre of intercanine tip space deficiency, approximately 0.6 mm of intercanine expansion occurs. Conversely, correcting a calculated intercanine tip

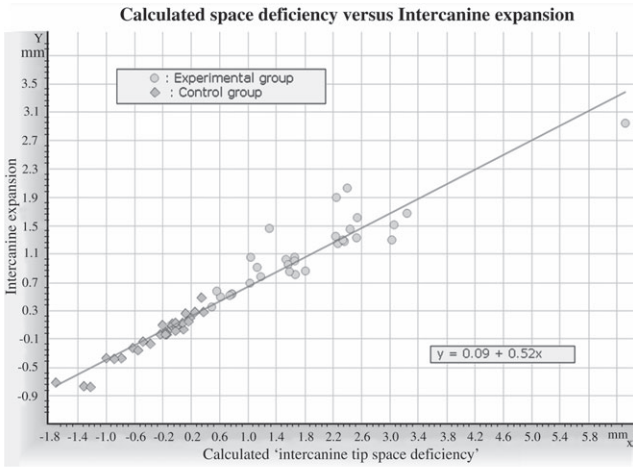

Figure 4 Regression line showing the relationship between the calculated intercanine tip space deficiency (x) and the intercanine expansion (y).

space deficiency of 1.75 mm would theoretically lead to an increase of about 1 mm in intercanine distance. The only way to prevent such an increase would be to either strip interproximally or to compress the aligned intercanine width, leading to an increase in arch depth and proclination of the incisors (Braun and Hnat, 1997).

Although the aforementioned intercanine tip space deficiency may not constitute a measure of total anterior crowding, for this project it was preferred over measuring crowding in between the distal anatomic contact points of the canines. This was done for three reasons: first, since intercanine width was measured at the canine tips and the canine tips were aligned on the curve, it seemed logical to calculate the space deficiency in between these canine tips to correlate to the measured intercanine expansion. Secondly, as only patients with relatively favourable canine positions were included, the canine tips were generally also quite favourably positioned and seemed to be less influenced by (de)rotation in comparison with the distal anatomic contact points. Finally, due to the curved nature of the canines and first premolars, aligning the mesial and distal anatomic contact points on the curve will often place the canine tip buccal to the curve. Instead, aligning the mesial anatomic contact point and tip of the canine on the curve will allow the distal anatomic contact points of the canine to meet the more palatally located mesial anatomic contact point of the first premolar.

The patient represented at the far right corner of Figure 4 is a somewhat special case, as an exceptionally high intercanine tip space deficiency of 6.4 mm was found. For most patients, this would seem to be beyond the scope of the leeway space to absorb. In fact, only approximately 4.3 millimetre of leeway space was found in this particular case. However, since the residual lack of space was reasonable (in this case, 2.1 mm), and could be 'stripped out' laterally at the level of the premolars, distal movement of the canines could theoretically and practically still take place, and it was therefore decided to include this patient in the study.

Of course, in representing a two-dimensional simplification of a complex three-dimensional situation, the present model has its limitations, for instance, canine tip was not corrected. It should be remembered, however, that only patients with a relatively favourable canine tip were included. Furthermore, since it is highly unlikely that any such required change in canine tip would consistently be in the same direction, the possible effects on space requirements, and hence on intercanine expansion, should be random and averaged out throughout the patient sample.

Also, changes in incisor inclination (torque) were not accounted for (O'Higgins *et al.*, 1999; Kirschen *et al.*, 2000). Nevertheless, by including only patients with a favourable incisor inclination, with leeway space present, no significant changes in incisor inclination would be expected or desired. Finally, due to the proximity of the incisal edge to the interproximal contact points of the mandibular teeth, minor changes in incisor inclination would not significantly influence space requirements (Kirschen *et al.*, 2000).

The effects of correcting the curve of Spee were not included in the model as its correction would merely add to the posterior space requirements, and therefore not diminish the amount of canine distalization required for the correction of the anterior space deficiency. Additionally, the effects themselves are somewhat controversial; as a general rule of thumb, it is usually stated that for every millimetre of curve of Spee to be corrected, 1 mm of space needs to be added to the space requirements for the left and right lateral segments. Some recent studies questioned the amount of space needed to correct the curve of Spee (Germane *et al.*, 1992; Braun *et al.*, 1996; Kirschen *et al.*, 2000), stating that it is mainly tooth morphology or the biomechanics involved, including the force system and wire types, that determine the various effects frequently observed during curve of Spee correction.

Conclusions

A significant increase in intercanine distance was found when distalizing the canines into a wider part of the mandibular alveolar process during the correction of crowding using the leeway space. High correlation coefficients were found when using three functions combined to mathematically describe the natural arch form, accurately reproducing the original intercanine width, intermolar width, and arch depth. A regression formula was constructed describing the relationship between the calculated intercanine tip space deficiency (x) and the intercanine expansion (y): $y = 0.09 + 0.52x$.

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References

BeGole E A 1980 Application of the cubic spline function in the description of dental arch form. Journal of Dental Research 59: 1549–1556

Braun S, Hnat W P 1997 Dynamic relationships of the mandibular anterior segment. American Journal of Orthodontics and Dentofacial Orthopedics 111: 518–524

Braun S, Hnat W P, Fender D E, Legan H L 1998 The form of the human dental arch. Angle Orthodontist 68: 29–36

Braun S, Hnat W P, Johnson B E 1996 The curve of Spee revisited. American Journal of Orthodontics and Dentofacial Orthopedics 110: 206–210

Brennan M M, Gianelly A A 2000 The use of the lingual arch in the mixed dentition to resolve incisor crowding. American Journal of Orthodontics and Dentofacial Orthopedics 117: 81–85

Burke S P, Silveira A M, Goldsmith L J, Yancey J M, Van Stewart A, Scarfe W C 1998 A meta-analysis of mandibular intercanine width in treatment and postretention. Angle Orthodontist 68: 53–60

Currier J H 1969 A computerized geometric analysis of human dental arch form. American Journal of Orthodontics 56: 164–179

Dugoni S A, Lee J S, Varela J, Dugoni A A 1995 Early mixed dentition treatment: postretention evaluation of stability and relapse. Angle Orthodontist 65: 311–320

Felton J M, Sinclair P M, Jones D L, Alexander R G 1987 A computerized analysis of the shape and stability of mandibular arch form. American Journal of Orthodontics and Dentofacial Orthopedics 92: 478–483

Gardner S D, Chaconas S J 1976 Posttreatment and postretention changes following orthodontic therapy. Angle Orthodontist 46: 151–161

Germane N, Staggers J A, Rubenstein L, Revere J T 1992 Arch length considerations due to the curve of Spee: a mathematical model. American Journal of Orthodontics and Dentofacial Orthopedics 102: 251–255

Hnat W P, Braun S, Chinhara A, Legan H L 2000 The relationship of arch length to alterations in dental arch width. American Journal of Orthodontics and Dentofacial Orthopedics 118: 184–188

Jones M L, Richmond S 1989 An assessment of the fit of a parabolic curve to pre- and post-treatment dental arches. British Journal of Orthodontics 16: 85–93

Kirschen R H, O'Higgins E A, Lee R T 2000 The Royal London space planning: an integration of space analysis and treatment planning: Part I: assessing the space required to meet treatment objectives. American Journal of Orthodontics and Dentofacial Orthopedics 118: 448–455

Lestrel P E, Takahashi O, Kanazawa E 2004 A quantitative approach for measuring crowding in the dental arch: Fourier descriptors. American Journal of Orthodontics and Dentofacial Orthopedics 125: 716–725

Little R M, Riedel R A, Stein A 1990 Mandibular arch length increase during the mixed dentition: postretention evaluation of stability and relapse. American Journal of Orthodontics and Dentofacial Orthopedics 97: 393–404

Nance H N 1941 The limitation of orthodontic treatment: I, mixed dentition diagnosis and treatment. American Journal of Orthodontics 47: 177–227

Noroozi H 2000 A formula to determine the amount of retraction of mandibular canines. Angle Orthodontist 70: 154–156

Noroozi H, Nik T H, Saeeda 2001 The dental arch form revisited. Angle Orthodontist 71: 386–389

O'Higgins E A, Kirschen R H, Lee R T 1999 The influence of maxillary incisor inclination on arch length. British Journal of Orthodontics 26: 97–102

Pepe S H 1975 Polynomial and catenary curve fits to human dental arches. Journal of Dental Research 54: 1124–1132

Sampson P D 1981 Dental arch shape: a statistical analysis using conic sections. American Journal of Orthodontics 79: 535–548

Wellens H 2007 Applicability of mathematical curve-fitting procedures to late mixed dentition patients with crowding: a clinical-experimental evaluation. American Journal of Orthodontics and Dentofacial Orthopedics 160: e17–25